Conservative Forces

If work done by a force is independent of path followed, the force is called Conservative

$\Rightarrow$ Work done along closed path = zero.

$$\boxed{\oint \vec{F}.d\vec{l} = 0} \qquad \{F \text{ is conservative}\}$$

Using stoke's theorem to convert into surface integral.

$$\int (\vec{\nabla} \cdot \vec{F}) . d\vec{s} = 0$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{F} = 0} \quad \text{--- (i)}$$

From mathematics

$$\pm \vec{\nabla} \times \vec{\nabla} \text{ (scalar) } = 0 \quad \text{--- (ii)}$$

Calculating (i) & (ii)

$$F = \pm \vec{\nabla} \text{ scalar}$$

Or

F from U $\rightarrow$ $\boxed{F = -\nabla U}$ — ve sign gives current direction of force.

$$U = P.E.$$

It is defined only for conservative forces.
If we fined U from F

Actual definition $\rightarrow$ $\boxed{U(r) = \int_{r}^{ref.pt.} \vec{F}.d\vec{l}}$

Ref. pt. the pt. where P.E. is zero & also force is zero.

Gravitational Force [$F = mgh$ is used when $h \lll R_e$]

Short Method :-

$$U(x) = -\int f\,dx$$

$$U(r) = -\int f\,dr$$

If force is repulsive it is taken +ve
If force is attractive it is taken -ve.

Example of conservative forces :-

1. Gravitational force b/w two masses :-

$$\boxed{f = \frac{G\,m_1 m_2}{r^2}}$$ (attractive)

{ -ve sign is taken because force is attractive

$$U = -\int f.dr = -\int \frac{-G m_1 m_2}{r^2}\,dr = -\frac{G m_1 m_2}{r}$$

2. Potential Energy of Earth Mass System :-

$$U = -\frac{G M_e m}{R_e + h}$$

P.E. = 0 at surface with $h = 0$

$$\Rightarrow \frac{-G M_e m}{R_e\left(1+\frac{h}{R_e}\right)} = \frac{-G M_e m}{R_e}\left(1+\frac{h}{R_e}\right)^{-1}$$

P.E. $= U_{ref.} = -\dfrac{G M_e m}{R_e}$

$$\Rightarrow U = -\frac{G M_e m}{R_e}\left(1-\frac{h}{R_e}\right)$$

$$\Rightarrow U = -\frac{G M_e m}{R_e} + \frac{G M_e m}{R_e^2}h$$

{ $\because U_{ref.} = -\dfrac{G M_e m}{R_e}$, $g = \dfrac{G M_e}{R_e^2}$

$$\Rightarrow U = U_{ref} + mgh$$ (true for small h) $h \lll R_e$

$$\Rightarrow U - U_{ref} = mgh$$

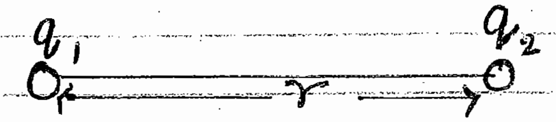
{ If $h = 2R_e$ or $\frac{R_e}{2}$ etc then this relation will not be used }

To convert electrostatic formulae into gravitational formulas
Replace $q \longrightarrow m$, & $\frac{1}{4\pi\epsilon_0} \longrightarrow G$

if $U_{ref} = 0$

Then, $\boxed{U = mgh}$

3. Electrostatic force :- {It is a conservative force.}

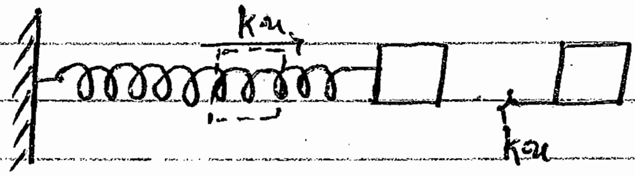


$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$q_1 > 0$, $q_2 > 0$

$$U = -\int F.\,dr \Rightarrow \boxed{U = \frac{q_1 q_2}{4\pi\epsilon_0 r}}$$

4. Spring force {Conservative} :-



$$U(x) = -\int F\,dx$$
$$= -\int (-kx)\,dx$$
$$\boxed{U(x) = \frac{1}{2}kx^2}$$

* Conservation of Mechanical Energy :-

If only conservative forces are acting on a system then total mechanical energy of the system remains conserved.

$$\boxed{\text{Total Energy} = \text{Constant}}$$

$$K.E. + P.E. = \text{Constant}$$

$$(K.E. + P.E.)_i = (K.E. + P.E.)_f.$$

Mechanical energy is not conserved if non conservative forces act on the system.

Non-Conservative forces → friction and any other dissipative forces.

{If there is other than above three forces} then we use Work Energy Theorem}.

* Work Energy Theorem :-

Total work done by all forces is equal to change in K.E.

$$\boxed{\text{Total Work Done} = K.E_f - K.E_i}$$